ABSTRACT

of the dissertation for the degree of Doctor of Philosophy (PhD)
specialty «6D070300 – Information systems»
titled «**Construction of hybrid transforms to solve signal processing problems in systems**»
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The topic of the dissertation research is the construction of hybrid transforms based on classical transforms to solve the problems of signal processing in systems. The foundation of all transformations is the Fourier transform (FT). FT is found in all areas of engineering, so it is important to develop and understand its various aspects. FT describes stationary signals. Natural signals (audio, video, biomedical, linear-frequency-modulated (LFM), etc.) are not stationary, and FT is not sufficient for such signals. Linear canonical transform (LCT) and its general form offset linear canonical transform (OLCT) are proposed for studying non-stationary signals. OLCT has five and LCT has three free parameters. This makes them more flexible compared to fractional Fourier transform (FrFT) with one free parameter and FT with no free parameter. LCT has various applications in various areas. For example, in optics, a wide class of optical systems can be modeled by LCT. Also, LCT is useful in many signal processing applications. For example, in time-frequency analysis, filter design, signal synthesis, phase reconstruction, encryption, radar and sonar systems, etc. In fact, the properties and applications of OLCT are similar to LCT, but they are more general than LCT. For example, LCT can model optical systems but cannot analyze with prisms or moving lenses, while OLCT can both model optical systems and analyze with prisms or moving lenses.

It has been proven that the Wigner-Ville distribution (WVD) is a special type of quasi-probability distribution. WVD plays a major role in signal time-frequency analysis and signal processing. Among many known time-frequency analyses, the WVD is an important and valuable technique that can provide high precision in LFM signal detection and parameter estimation. The LFM signal is used in communications, radar and sonar systems. Hence, LFM signal detection and estimation is undoubtedly one of the most important topics in engineering. WVD and LCT/OLCT transformations are used in LFM signal processing, but they have following disadvantages:

- WVD does not fully exploit the phase feature of LFM signal;
- LCT/OLCT cannot gather signal energy strongly like WVD.

This results in poor performance under a low signal-to-noise ratio for detection and estimation. Solving this problem is an extensive part of the research of the dissertation work.

It is known that the signals encountered in life are non-stationary or time-varying, that is, unstable. In addition to WVD, LCT, OLCT, short-time Fourier transform (STFT), wavelet transform (WT) were defined to study such signals. Compared to STFT, WT describes the signal better. However, WT is not without flaws. In the high frequency region, the frequency resolution of WT is poor. Elimination of such shortcomings is the second part of dissertation research.

The aim of the research is to construct hybrid transforms, such as Wigner-Ville distribution in the offset linear canonical transform (WOL) and quadratic phase wavelet packet transform (QP-WPT) for solving signal processing problems in systems and to prove their main properties. And to show that these hybrid transforms can be used in

signal detection and signal reconstruction. The general goal of the dissertation is to combine signal processing tools without losing advantages of both classical transforms and construct a new hybrid transforms that have better results in applications compared to classical ones and are less computationally complex. Then, further expand the quaternion FT by bridging quaternionic theory with probability to open avenues for advanced applications in signal processing, communications, information systems, and applied mathematics, potentially driving significant advancements in these fields.

Research objectives. There is some complexity in the theory and application of WVD-OLCT. For example: the WVD-OLCT formula is long, so it is inconvenient and time-consuming to enter into a computer. In order to overcome these complexities, we introduce a new formulation for WVD based on OLCT. It will be a simpler time-frequency analysis tool than WVD-OLCT. One of the tasks of the research work is to describe a number of basic properties of the newly obtained transforms, including marginal properties, Moyal's formula, uncertainty principle, etc. In addition, we propose a new method of calculating the instantaneous frequency. Then, we apply the proposed transform to signal reconstruction and detection. In general, one of the main objectives of this research is to define a hybrid tool for signal processing, which has a low computational cost and a simple formulation, but which is not inferior to classical WVD, WVD-OLCT in terms of properties and applications. Another objective of this research is to study the theory of QP-WPT that is constructed based on WPT and QPFT. Establish the QP-WPT relation with other transforms and prove the main properties and theorems. In addition, we will show signal reconstruction by QP-WPT as an application of QP-WPT.

Research methods. In this dissertation, based on the theory of classical transforms, hybrid transforms were constructed using theoretical and mathematical methods. By the analysis research method, special cases of hybrid transforms were described, various properties were proved, several applications were shown, and connections with other transformations were also given.

Basic concepts proposed for defense (proved scientific hypotheses and other conclusions that are new knowledge). The new results of this research work are shown below:

- Construction of a new type of WVD-OLCT called WOL;
- Construction of a new signal processing tool called QP-WPT;
- Introduction of a quaternion fractional Fourier transform (1DQFRFT);
- Prove the main and important properties of WOL and QP-WPT;
- Establishment of connection between FT and WFT with QP-WPT;
- Show several types of uncertainty inequalities associated with QP-WPT;
- Demonstrate the applications of WOL;
- Signal reconstruction using QP-WPT
- Explore the 1DQFRFT’s unique applications to probabilistic methods, particularly for modeling and analyzing stochastic processes.

The development of hybrid transforms is an innovative and research-worthy. The study of signal processing techniques helps to gain a greater understanding of the applications of transforms and to transfer knowledge gained from one subject to another.

Description of the main results of the research. This dissertation researches hybrid transforms and mainly considers WOL, which incorporates the advantages of OLCT and the properties of WVD. It is a simpler signal processing tool than WVD-OLCT and WVD. We have discussed some basic properties of WOL in this research,

including marginal properties, Moyal's formula, and the uncertainty principle. In addition, a new calculation method of instantaneous frequency was described. Several applications of the newly proposed hybrid transform are also shown.

Based on the theory of QPFT and classical WPT, in this dissertation we proposed a new signal processing tool called QP-WPT, which overcomes the deficiencies of WPT and QPFT. In general, it is not only combines the advantages of QPFT and WPT, but also retains the properties and has better mathematical properties. In addition to exploring some important fundamental properties, including Moyal's formula, inversion formula, and reproductive kernel properties, we formulated several classes of uncertainty inequalities, such as Leib's uncertainty principle, logarithmic uncertainty inequality, and Heisenberg's inequality. Finally, we have shown that the signal can be reconstructed using QP-WPT.

Novelty and significance of the obtained results. The definitions, properties, applications, conclusions, and recommendations obtained in this dissertation develop and complement a number of aspects of signal processing theory. And it contributes to the formation of ideas for the implementation of various new applications. The main results and conclusions contained in the dissertation work can be used in various signal processing applications, image detection, as well as solving a number of problems in mathematics, optics and engineering.

Compliance with the directions of development of science or government programs. Some of the research works described in the dissertation was carried out and is continuing within the framework of the project "Construction and study of quaternion Fourier transforms and their application for creation of information systems for geophysics and geochemistry problems", which won the competition for grant funding of fundamental and applied scientific research for scientific and scientific-technical projects for 2022-2024 aimed at implementing the program documents of the Republic of Kazakhstan. Competition name: Ministry of Education and Science of the Republic of Kazakhstan competition for grant funding for scientific and (or) scientific and technical projects for 2022-2024; Project No. AP14871252. The project is included in the priority direction of science development of the Republic of Kazakhstan, namely "Information, communication and space technologies". The author held the position of "Researcher" in the research group of the mentioned project. The project successfully implemented at the National Engineering Academy of the Republic of Kazakhstan.

Description of the doctoral student's contribution to the preparation of each publication. 5 scientific works with a total volume of 11,75 printed pages were published on the topic of the dissertation.

The results of WT-related research have been successfully peer-reviewed and published by publisher Elsevier in the well-known and trusted German scientific journal "Optik":

1) *Quadratic-phase wave packet transform*, Optik, 261: 2022. (Cited 22 times) WoS: Q2; Scopus: Engineering Q2, Rank 189/738, Percentile 77%.

Author's contribution statement CRediT (Contributor Roles Taxonomy) in writing the article: conceptualization (the idea of the article belongs to the author; the author has constructed a new transform based on the WT and QPFT, i.e. defined QP-WPT and formulated its properties and applications), formal analysis (the author proved the main properties of QP-WPT by analyzing and using mathematical methods), writing — writing the original article (the author prepared the first draft of the manuscript in LaTeX), writing — reviewing and editing (the author was involved in writing a response

for anonymous reviewers comments appointed by the journal's editors and revising the manuscript before publication).

Part of the research results on WVD and OLCT were presented at a peer-reviewed international conference in Thailand and published in the Advances in Intelligent Systems Research series by Atlantis Press, a division of Springer Nature:

2) *The Wigner-Ville distribution based on the offset linear canonical transform domain*, Proceedings of the 2nd International Conference on Modelling, Simulation and Applied Mathematics (MSAM2017), 139–142, 2017 (Cited 11 times).

The author's contribution CRediT in the writing of the 2)-article: conceptualization (the idea of the article belongs to the author; the author formulated the objects of research to construct a compact hybrid transformation by combining WVD and OLCT, and demonstrated its main properties and applications), methodology (the author formulated the structure of the article), formal analysis (the author used mathematical methods to analyze the research), writing — writing the original article (the author wrote and prepared the initial draft of the published work), writing — reviewing and editing (the author wrote a response for anonymous reviewers comments appointed by the journal's editors), funding (the conference participation fee was financed by the author).

Additionally, in 2023, another part of the research results on WVD and OLCT have been published as a book chapter:

3) Chapter: *Hybrid Transforms*. In book: Time Frequency Analysis of Some Generalized Fourier Transforms. IntechOpen; 2023. 10.5772/intechopen.108186.

Statement of the author's contribution to the writing of one of the six chapters of the book which is written under the editorial guidance of a foreign scientific consultant: conceptualization (the idea of the chapter belongs to the author; the author formulated the object and goals of the study), methodology (the author developed and designed the structure of the chapter), formal analysis (the author analyzed the research by using mathematical methods of solving integral equations, investigation (proved the properties of the hybrid transforms and showed its theoretical application), writing — writing the original chapter (the author wrote the initial and final published work in accordance with the requirements), writing — review and editing (author wrote response, made corrections according to the reviews), funding (funded by the Scientific Committee of the Ministry of Education and Science of the Republic of Kazakhstan, No. AP14871252, within the framework of the project "Construction and study of quaternion Fourier transforms and their application for creation of information systems for geophysics and geochemistry problems ").

The results on bridging signal processing and probability theory in the framework of 1DQFRFT have been successfully peer-reviewed and published by "Mathematics" journal:

4) *Quaternion fractional Fourier transform: bridging signal processing and probability theory*, Mathematics, 13(2), 195, 2025. WoS: Q1, Rank 21/490, Percentile 97,45%; Scopus: Computer Science (miscellaneous) Q2, Rank 48/133, Percentile 64%.

The author's contribution CRediT in the writing of the 4)-article: writing — review and editing (the author participated in writing a reply on the critical reviews of three anonymous reviewers appointed by the journal and in the revision and editing of the manuscript before publication, prepared responses to reviews related to the use of quaternion signal processing methods in solving probability theory problems, and made appropriate corrections and additions to the manuscript), funding (funded by the Scientific Committee of the Ministry of Education and Science of the Republic of

Kazakhstan, No. AP14871252, within the framework of the project "Construction and study of quaternion Fourier transforms and their application for creation of information systems for geophysics and geochemistry problems ").

The results on hybrid and quaternion transforms have been successfully published as a monograph:

5) *Кватернионные преобразования Фурье и их применение в задачах геофизики и геохимии*, Monograph, Almaty: Everest, 2024. – 140 p. ISBN 978-601-04-6809-2.

Statement of the author's contribution to the writing of the monograph, written under the editorial guidance of the project leader: conceptualization (an immense part of the idea of the monograph belongs to the author; the author participated in the processing of signals, images, and geophysical modeling problems using quaternion Fourier transforms), methodology (the author developed and designed the structure of the monograph), formal analysis (the author used research in the field of signal processing, geoinformation systems, and applied mathematics), investigation (the author combined the advantages of various methods and increased the accuracy of data processing), writing — writing the original monograph (the author was involved in developing and writing the original monograph in accordance with the requirements), writing — reviewing and editing (the author made corrections in accordance with the comments of reviewers), funding (funded by the Scientific Committee of the Ministry of Education and Science of the Republic of Kazakhstan, No. AP14871252, within the framework of the project "Construction and study of quaternion Fourier transforms and their application for creation of information systems for geophysics and geochemistry problems ").